II. CONSTITUTIVE RELATIONS

2.1 General Considerations

The term "constitutive relations" is used in a generic sense to encompass all material properties which must be combined with the equations of continuity, motion, and energy conservation to supply a complete set of flow equations. Constitutive relations in practice usually reduce to an equation of state relating pressure, volume, and temperature or internal energy. This simplification is partially enforced by ignorance of other material relations; it often yields, in addition, quite a good description of wave propagation over a wide range of parameters. The equation of state is necessarily accompanied by a statement about the variation of specific heat with pressure and temperature. It may on occasion include information about rigidity and yield or even rate effects. For the present we consider the equation of state as defined above. These considerations are themselves useful and they will provide insight into the requirements which must be satisfied by more general constitutive relations. The following remarks are necessarily limited in their scope. More detailed information will be found in various review articles (1,2).

2.2 Equations of State for Fluids (3)

A "complete" equation of state for a fluid is a relation between thermodynamic variables which is sufficient for calculating any thermodynamic parameter of the material, given two. For example, if specific internal energy, E, is given as a function of specific entropy, S, and specific volume v, $E = E(S,v)$, then

$$p = -(\partial e/\partial v)_S$$
$$T = (\partial e/\partial S)_v$$
$$H = E + pv$$
etc.,

where p, T, H are pressure, temperature and specific enthalpy. On the other hand, if pressure is given as a function of T and v, as usually occurs, then neither internal energy nor enthalpy can be calculated without specifying the specific heat.

When partial equations of state are given, as in the last example, then certain limitations are placed on other thermodynamic quantities if all are to be compatible. This is illustrated by the following example. Suppose that specific heat at constant pressure is known as a function of temperature and assumed to be a function of temperature alone: $C_p = C_p(T)$. Then by the following argument we can see that the relation between p, v, and T must be that of Eq. (2.1) below:

$$(\partial H/\partial T)_p = C_p(T) \quad \text{by definition.} \tag{2.1}$$

If $H = H(p,T)$, then

$$dH = C_p dT + (\partial H/\partial p)_T \, dp$$

or

$$H = \int C_p(T)dT + f(p) .$$